

Q1: Let $R \rightarrow R$ be defined by

$$f(x) = \frac{2x+1}{2x+2}$$

Is f one-to-one?

5marks

Solution:

$$\frac{2x_1+1}{2x_1+2} = \frac{2x_2+1}{2x_2+2}$$

$$(2x_1+1)(2x_2+2) = (2x_2+1)(2x_1+2)$$

$$4x_1x_2 + 4x_1 + 2x_2 + 2 = 4x_1x_2 + 4x_2 + 2x_1 + 2$$

$$\cancel{4x_1x_2} + 4x_1 + 2x_2 + \cancel{2} - \cancel{4x_1x_2} - 4x_2 - 2x_1 - \cancel{2} = 0$$

$$4x_1 + 2x_2 - 4x_2 - 2x_1 = 0$$

$$4x_1 - 2x_1 = 4x_2 - 2x_2$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

$x_1 = x_2$ Therefore this function is one to one function

Q2: Find the gcd of 1075, 45 using dividing algorithm.

5marks

Solution:

1. Divide 1075 by 45:

$$\text{This gives } 1075 = 45 \cdot 23 + 40$$

2. Divide 45 by 40:

$$\text{This gives } 45 = 40 \cdot 1 + 5$$

3. Divide 40 by 5:

$$\text{This gives } 40 = 5 \cdot 8 + 0$$

Hence $\text{gcd}(1075, 45) = 5$.

Q 3. Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for $x=-2.01$

3 marks

Solution: Page 249

$$\lfloor -2.01 \rfloor = \lfloor -3 + 0.99 \rfloor = -3$$

$$\lceil -2.01 \rceil = \lceil -3 + 0.999 \rceil = -3 + 1 = -2$$

Q 4: Find the greatest common division for the following pair of Integer: 30,105 2marks

Solution:

1. Divide 105 by 30:

$$\text{This gives } 105 = 30 * 5 + 0$$

Hence $\text{gcd}(105,30) = 30$

Q 5: Find the Spanning tree for the graph $K_{1,5}$?

2marks

Solution: Page 332

$K_{1,5}$ represents a complete bipartite graph on (1,5) vertices, drawn below:



Clearly the graph itself is a tree (six vertices and five edges). Hence the graph is itself a spanning tree.

Q 6 Determine which f is a function?

$$f(x) = \frac{1}{x^2 - 4}$$

Q7. What is the difference between $\{a,b\}$ and $\{\{a,b\}\}$?

Solution:

$\{a,b\}$ is a set while $\{\{a,b\}\}$ is a subset of some set.

Q8. How many 3-digits can be formed by using each one of the digits 2,3,5,7,9 only once?

Solution:

$$5 * 4 * 3 = 60$$

Q9 what is the smallest integer N such that $\lfloor N/9 \rfloor = 6$?